



# Cambridge International AS & A Level

CANDIDATE  
NAME

--

CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--



## FURTHER MATHEMATICS

9231/32

Paper 3 Further Mechanics

May/June 2023

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ( $g$ ) is needed, use  $10 \text{ ms}^{-2}$ .

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 One end of a light elastic string, of natural length  $a$  and modulus of elasticity  $3mg$ , is attached to a fixed point  $O$ . The other end of the string is attached to a particle  $P$  of mass  $m$ . The string hangs with  $P$  vertically below  $O$ . The particle  $P$  is pulled vertically downwards so that the extension of the string is  $2a$ . The particle  $P$  is then released from rest.

(a) Find the speed of  $P$  when it is at a distance  $\frac{3}{4}a$  below  $O$ . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the initial acceleration of  $P$  when it is released from rest. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

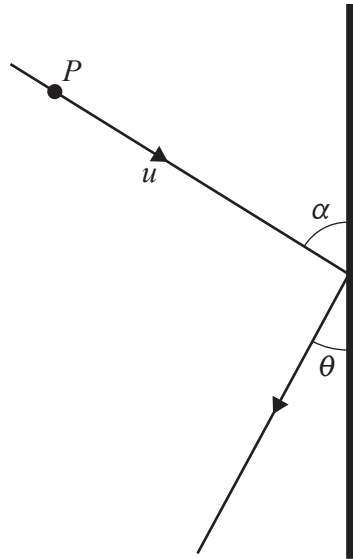
.....

.....

.....

.....

.....



A particle  $P$  of mass  $m$  is moving with speed  $u$  on a fixed smooth horizontal surface. It collides at an angle  $\alpha$  with a fixed smooth vertical barrier. After the collision,  $P$  moves at an angle  $\theta$  with the barrier, where  $\tan \theta = \frac{1}{2}$  (see diagram). The coefficient of restitution between  $P$  and the barrier is  $e$ . The particle  $P$  loses 20% of its kinetic energy as a result of the collision.

Find the value of  $e$ .

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3 A particle  $P$  of mass  $m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point  $O$ . The particle  $P$  is held at the point  $A$ , where  $OA$  makes an angle  $\theta$  with the downward vertical through  $O$ , and with the string taut. The particle  $P$  is projected perpendicular to  $OA$  in an upwards direction with speed  $u$ . It then starts to move along a circular path in a vertical plane. The string goes slack when  $P$  is at  $B$ , where angle  $AOB$  is  $90^\circ$  and the speed of  $P$  is  $\sqrt{\frac{4}{5}ag}$ .

(a) Find the value of  $\sin \theta$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find, in terms of  $m$  and  $g$ , the tension in the string when  $P$  is at  $A$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

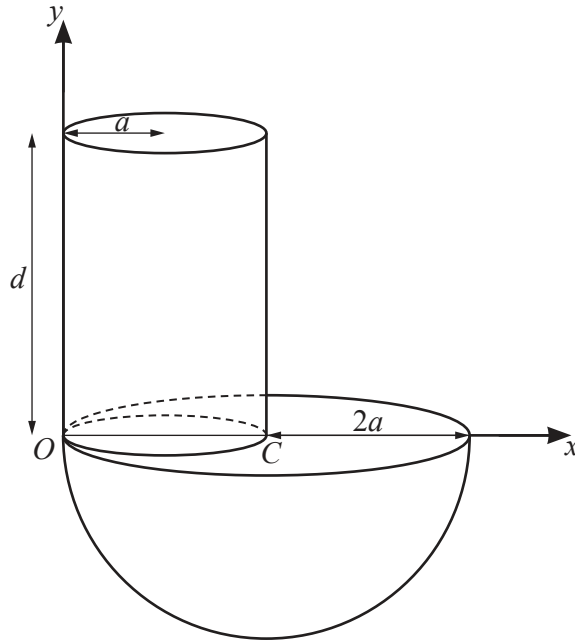
.....

.....

.....

.....

A series of 20 horizontal dotted lines for writing.



An object is formed from a solid hemisphere, of radius  $2a$ , and a solid cylinder, of radius  $a$  and height  $d$ . The hemisphere and the cylinder are made of the same material. The cylinder is attached to the plane face of the hemisphere. The line  $OC$  forms a diameter of the base of the cylinder, where  $C$  is the centre of the plane face of the hemisphere and  $O$  is common to both circumferences (see diagram). Relative to axes through  $O$ , parallel and perpendicular to  $OC$  as shown, the centre of mass of the object is  $(\bar{x}, \bar{y})$ .

- (a) Show that  $\bar{x} = \frac{32a^2 + 3ad}{16a + 3d}$  and find an expression, in terms of  $a$  and  $d$ , for  $\bar{y}$ . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



- 5 A light elastic string of natural length  $a$  and modulus of elasticity  $\lambda mg$  has one end attached to a fixed point  $O$  on a smooth horizontal surface. When a particle of mass  $m$  is attached to the free end of the string, it moves with speed  $v$  in a horizontal circle with centre  $O$  and radius  $x$ . When, instead, a particle of mass  $2m$  is attached to the free end of the string, this particle moves with speed  $\frac{1}{2}v$  in a horizontal circle with centre  $O$  and radius  $\frac{3}{4}x$ .

(a) Find  $x$  in terms of  $a$ .

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....









7 At time  $t$  s, a particle  $P$  is projected with speed  $40 \text{ m s}^{-1}$  at an angle  $\theta$  above the horizontal from a point  $O$  on a horizontal plane and moves freely under gravity. The greatest height achieved by  $P$  during its flight is  $H$  m and the corresponding time is  $T$  s.

(a) Obtain expressions for  $H$  and  $T$  in terms of  $\theta$ . [2]

.....

.....

.....

.....

.....

.....

During the time between  $t = T$  and  $t = 3$ ,  $P$  descends a distance  $\frac{1}{4}H$ .

(b) Find the value of  $\theta$ . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the speed of  $P$  when  $t = 3$ .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....





**BLANK PAGE**

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at [www.cambridgeinternational.org](http://www.cambridgeinternational.org) after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.